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Field theoretical calculation of the specific heat exponent for a classical N-vector model in a random external field

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Abstract. We calculate using diagrammatic perturbation theory in the two-loop approximation, the specific heat exponent α for the classical N-vector model in a random external field for spatial dimension (D) lying between four and six. The calculation supports the modified hyperscaling $(D-2)\nu = 2-\alpha$, where ν is the correlation length exponent.

PACS. 75.40.-s Critical-point effects, specific heats, short-range order – 75.40.Cx Static properties (order parameter, static susceptibility, heat capacities, critical exponents, etc.)

1 Introduction

The study of random field systems, specially random field Ising models has been an exciting area of theoretical and experimental investigations [1]. We consider the question of the specific heat exponent in random systems. Unlike the other exponents, this exponent is generally not calculated directly, but is arrived at through the appropriate scaling relations. In general, it is the hyperscaling relation [2] ($(D-\theta)\nu = 2-\alpha$), which is frequently used since ν is the exponent which is easiest to calculate. In disordered systems, the randomness associated with the disorder usually overwhelms the temperature fluctuations [3,4] and thus there is an effective dimensionality, which comes into play. In general, it is this dimensional reduction [5] (θ) which one calculates and thus gets α from the modified hyperscaling relation. In this paper, we show for the classical N-vector model in a random external field that the usual diagrammatic tools can be used to calculate "α" directly and the modified hyperscaling is directly verified. The model and the correlation function for the specific heat are prescribed in Section 2, while the calculation of α is given in Section 3.

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2 The model and the specific heat

We begin with the continuum free energy functional for a N component field $\boldsymbol{\phi}$

$$\mathcal{F} = \int d^D x \left[\frac{r}{2} \sum_{i=1}^{N} \phi_i \phi_i + \frac{1}{2} (\boldsymbol{\nabla}\phi_i) . (\boldsymbol{\nabla}\phi_i) + \frac{u}{4} (\phi_i\phi_i)^2 - h_i\phi_i \right] \quad (1)$$

where ϕ_i denotes the i-th component of the N-component vector $\boldsymbol{\phi}_i$ and the field $\mathbf{h}$ distributed according to

$$P(\mathbf{h}) = \frac{1}{\sqrt{2\pi\Delta}} e^{-h^2/2\Delta} \quad (2)$$

and $r = T - T_0$ carrying the temperature dependence.

As usual, our task is to calculate the partition function which is given by

$$Z = \int dh P(h) \int \mathcal{D}[\phi] e^{-F}. \quad (3)$$

The randomness being frozen, we are required to actually do the thermal fluctuation calculation with the distribution of h fixed and then average over the distribution of h. The technical problem involved in this is surmounted by the so-called "replica trick" where one considers n replicas of the system under consideration and finally takes the limit of $n \to 0$. The quantity that one thus construct is Z^n and the limit $n \to 0$, ensures that the free energy

$$F = -\ln Z = -\lim_{n \to 0} \frac{1}{n} (Z^n - 1) \quad (4)$$

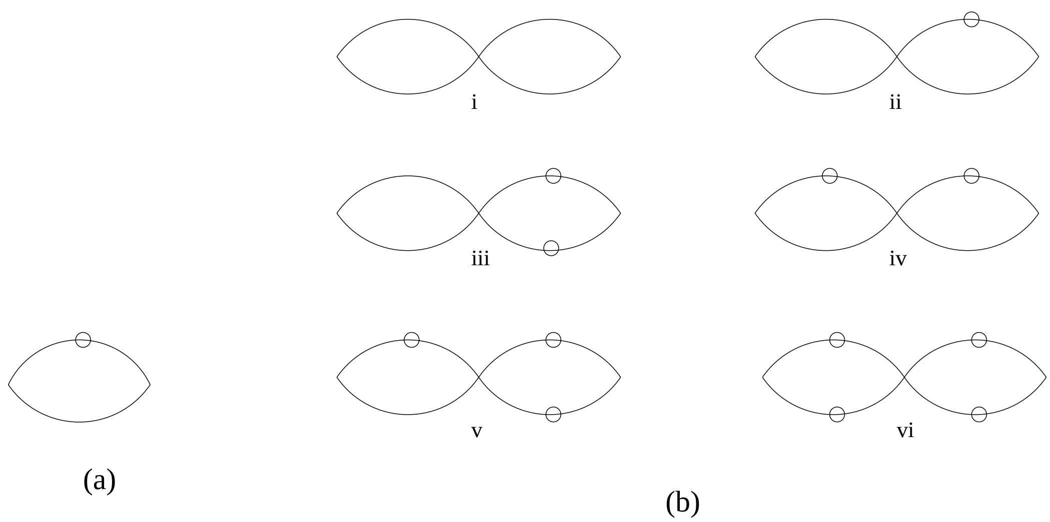

Fig. 1. The single loop contribution to the specific heat. The straight line denotes the propagator with $\Delta = 0$, while the circled line stands for the part of the propagator which is proportional to Δ. (b) The two loop contribution to the specific heat. The contribution of (i) and (ii) vanish since u is irrelevant, while the contributions of (iii), (v) and (vi) vanish in the limit $n \to 0$ because the replica combinatorics yields respective factors of n, n and n^2 for the three cases.

is correctly obtained. In the replicated partition function Z^n, the average over h can be performed and one is left with the calculation

$$Z^n = \int \mathcal{D}[\phi] e^{-A} \tag{5}$$

$$A = \int d^D x \left[\frac{r}{2} \phi_i^\alpha \phi_i^\alpha + \frac{1}{2} (\boldsymbol{\nabla} \phi_i^\alpha) . (\boldsymbol{\nabla} \phi_i^\alpha) + \frac{u}{4} (\phi_i^\alpha \phi_i^\alpha)^2 - \frac{\Delta}{2} \left(\sum_\alpha \phi_i^\alpha \right) \left(\sum_\beta \phi_i^\beta \right) \right]. \tag{6}$$

The specific heat is then given by $C \sim -\frac{\partial^2 F}{\partial T^2} \sim -\frac{\partial^2 F}{\partial r^2}$ and combining equations (4, 5, 6), we see

$$\begin{aligned} C &\sim \lim_{n\to 0} \frac{1}{n} \frac{\partial}{\partial r} \left[-Z^n \left\langle \int \frac{1}{2} \phi_i^\alpha \phi_i^\alpha d^D x \right\rangle \right] \\ &\sim \lim_{n\to 0} \frac{1}{4n} \left[Z^n \left\langle \int \phi_i^\alpha \phi_i^\alpha d^D x_1 \int \phi_j^\beta \phi_j^\beta d^D x_2 \right\rangle \right] \\ &\sim \lim_{n\to 0} \frac{1}{4n} \left\langle \int \phi_i^\alpha(x_1) \phi_i^\alpha(x_1) d^D x_1 \right. \\ &\quad \times \left. \int \phi_j^\beta(x_2) \phi_j^\beta(x_2) d^D x_2 \right\rangle. \end{aligned} \tag{7}$$

The correlation function above, although not explicitly connected, is actually a connected correlation function, since the disconnected part vanishes in the limit $n \to 0$.

3 Calculation of α

The basic ingredient of the calculation is the Gaussian correlation function [6]

$$G_{\alpha\beta}^{(o)}(k) = \frac{1}{r+k^2} \delta_{\alpha\beta} + \frac{\Delta}{(r+k^2)(r+k^2-n\Delta)} \cdot \tag{8}$$

In the Gaussian limit, the specific heat is simply (Fig. 1a)

$$\begin{aligned} C_{Gaussian} &\sim \lim_{n\to 0} \frac{V}{4n} \int d^D x_{12} \left\langle \phi^\alpha(x_1) \phi^\beta(x_2) \right\rangle \left\langle \phi^\alpha(x_1) \phi^\beta(x_2) \right\rangle \\ &\sim \lim_{n\to 0} \frac{V}{4n} \int d^D p \left\langle \phi^\alpha(p) \phi^\beta(-p) \right\rangle \left\langle \phi^\alpha(p) \phi^\beta(-p) \right\rangle \\ &\sim \lim_{n\to 0} \frac{V}{4n} \left[n \int d^D p \left[\frac{1}{p^2+r} + \frac{\Delta}{(p^2+r)^2} \right]^2 \right. \\ &\quad \left. + n(n-1)\Delta^2 \int \frac{d^D p}{(p^2+r)^3} \right] \\ &\sim \frac{V}{4} \left[\int \frac{d^D p}{(p^2+r)^2} + 2\Delta \int \frac{d^D p}{(p^2+r)^3} \right]. \end{aligned} \tag{9}$$

Since the lower critical dimension is 4 for the classical N-vector model in a random field [1], we need the above integral for $4 < D < 6$ (the upper critical dimension). In this above range the first integral is finite when an upper cut off is used. The long wavelength divergence comes from the second integral which consequently dominates for all D and we have

$$\frac{4C_{Gaussian}}{V} \sim \Delta \int \frac{d^D p}{(p^2+r)^3} \sim r^{(D-6)/2}. \tag{10}$$

Thus,

$$\alpha_{Gaussian} = \frac{6-D}{2} \cdot \tag{11}$$

From equation (8), we can read off the exponent ν as being 1/2 in the Gaussian limit and thus $(2-\alpha)/\nu = D-2$ giving rise to the hyperscaling relation

$$(D-2)\nu = 2 - \alpha. \tag{12}$$

We can now proceed beyond the Gaussian limit. Before, taking up the $\epsilon-$ expansion calculation ($\epsilon = 6 - D$), it is

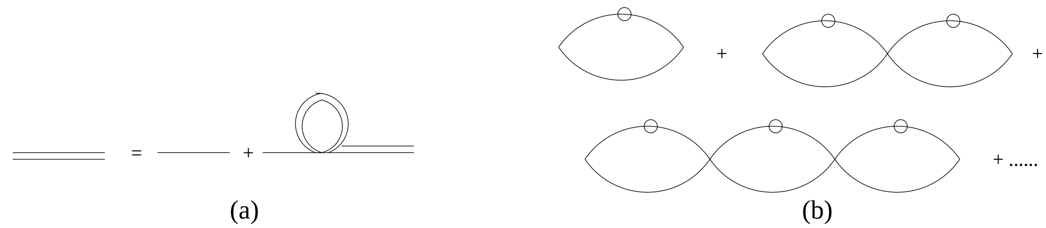

Fig. 2. (a) The propagator in the spherical limit. (b) The specific heat in the spherical limit.

instructive to see the answer in the spherical limit [7] *i.e.* $N \to \infty$. In the spherical limit [8],

$$G^{-1}_{\alpha\beta} = G^{-1}_{\alpha\beta}{}^{(0)} - \Sigma_{\alpha\beta}, \tag{13}$$

where the self energy $\Sigma(k)$ is given by (Fig. 2a)

$$\Sigma_{\alpha\beta} = \int d^D p G_{\alpha\beta}(p). \tag{14}$$

The integral is dominated by the second term and thus the shift in T_c is finite only for $D > 4$, which shows that $D = 4$ is the lower critical dimension. For $D > 4$, standard arguments now yield $\nu = \frac{1}{D-4}$ *i.e.* the spherical limit propagator is

$$G^{(s)}_{\alpha\beta}(k) = \frac{1}{m^2+k^2}\delta_{\alpha\beta} + \frac{\Delta}{(m^2+k^2)(m^2+k^2-n\Delta)} \tag{15}$$

where $m^2 \propto (T-T_c)^{2/(D-4)}$. The single loop (Fig. 1a) now scales as $(T-T_c)^{(D-6)/(D-4)}$ and the specific heat which is given by the buble sum (Fig. 2b) in the spherical limit is consequently given by [9] $A + B|T-T_c|^{(6-D)/(D-4)} +$ higher order terms, giving $\alpha = (D-6)/(D-4)$. Once again, $(2-\alpha)/\nu = D-2$, in agreement with the hyperscaling in equation (12).

To carry out a two loop calculation in the $\epsilon-$ expansion, we need to remember the following facts [10].

1. For $6 > D > 4$, the variable u is irrelevant *i.e.* $u \to 0$.
2. The combination $u\Delta$ is relevant for $D < 6$ and has a finite fixed point value, which controls the critical exponents for $6 > D > 4$.

Keeping the above in mind and doing the replica combinations first, we find that u, $u\Delta$, $u\Delta^3$ and $u\Delta^4$ terms disappear (we are comparing with the single loop term which is proportional to Δ). The two loop contribution to the specific heat consequently is

$$C \propto \Delta\left[\int \frac{d^D p}{(p^2+r)^3} - u\Delta(N+2)\left[\int \frac{d^D p}{(p^2+r)^3}\right]^2\right]. \tag{16}$$

Straightforward analysis now yields

$$\frac{\alpha}{\nu} = \frac{4-N}{N+8}\epsilon. \tag{17}$$

With $\nu = \frac{1}{2}(1+\frac{N+2}{2(N+8)}\epsilon)$, we again have $(2-\alpha)/\nu = D-2$ correct to $O(\epsilon)$.

4 Conclusion

Thus, we see that correct to $O(\epsilon)$, the dimensional reduction θ is 2. With the lower critical dimension working out to be $D = 4$, higher by 2 from the lower critical dimension of the normal system, $\theta = 2$ at $D = 4$ as well. This would lead one to suppose that $\theta = 2$ for $6 \geq D \geq 4$. However, the region close to $D = 4$, could be tricky.

We end by noting that the exponent β can be directly calculated from the correlation function using the scaling developed by Bray and Moore [2] and this, taken with our α and the known γ, yields the Rushbrooke equality $\alpha + 2\beta + \gamma = 2$ correct to $O(\epsilon)$ and in the spherical limit.

References

1. T. Nattermann, Cond-mat/9705295 and references therein; D.P. Belanger, A.P. Young, J. Magn. Mag. Mat. **100**, 272 (1991); H. Rieger, in *Annual Reviews of Computational Physics*, edited by D. Stauffer (World Scientific, 1995); *Spin Glasses and Random Fields*, edited by A.P. Young (World Scientific, 1998).
2. A.J. Bray, M.A. Moore, J. Phys. C. **18**, L927 (1985).
3. S. Fishman, A. Aharony, J. Phys. C. **12**, L729 (1979).
4. G. Grinstein, S.K. Ma, Phys. Rev. Lett. **49**, 685 (1982).
5. R.M. Hornreich, H.G. Schuster, Phys. Rev. B **26**, 3929 (1982).
6. J. Cardy, *Scaling and Renormalization in Statistical Physics* (Cambridge University Press, Cambridge, 1996).
7. T. Vojta, M. Schreiber, Phys. Rev. B **50**, 1272 (1994).
8. R. Abe, Prog. Theo. Phys. **48**, 1414 (1972); S.K. Ma, Phys. Rev. Lett. **29**, 1311 (1972).
9. R.A. Ferrell, D.J. Scalapino, Phys. Lett. A **41**, 371 (1972).
10. G. Grinstein, Phys. Rev. Lett. **37**, 944 (1976).